

# Easter revision

## Paper 1 Section A

*Each correct answer in this section is worth two marks.*

1. What is the distance, in units, between the points  $(-1, 2)$  and  $(4, 5)$ ?

- A.  $\sqrt{8}$
- B.  $\sqrt{16}$
- C.  $\sqrt{34}$
- D.  $\sqrt{58}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.1	C	0.64	0.5	NC	G1	HSN 054

$$\begin{aligned}
 \text{The distance is } & \sqrt{(4 - (-1))^2 + (5 - 2)^2} \\
 &= \sqrt{5^2 + 3^2} \\
 &= \sqrt{25 + 9} \\
 &= \sqrt{34}.
 \end{aligned}$$

Option ☒ C

2. What is the distance, in units, between the points  $(a, b)$  and  $(-b, a)$ ?

- A.  $\sqrt{2}\sqrt{a^2 + b^2}$
- B.  $\sqrt{2}(a + b)$
- C.  $\sqrt{2}(\sqrt{a} + \sqrt{b})$
- D.  $2\sqrt{a^2 + b^2}$

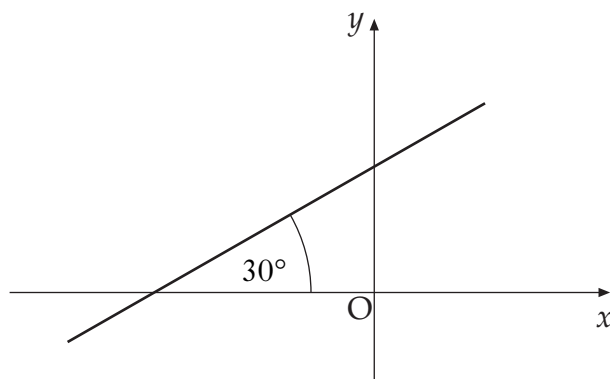
Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.1	C	0.3	0.23	CN	G1	HSN 011

$$\begin{aligned}
 d^2 &= (a - (-b))^2 + (b - a)^2 \\
 &= (a + b)^2 + (b - a)^2 \\
 &= a^2 + 2ab + b^2 + b^2 - 2ab + a^2 \\
 &= 2a^2 + 2b^2 \\
 &= 2(a^2 + b^2) \\
 d &= \sqrt{2(a^2 + b^2)} \quad \text{since } d \geq 0 \\
 &= \sqrt{2}\sqrt{a^2 + b^2}
 \end{aligned}$$

Remember:  
 $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Option A

3. A line makes an angle of  $30^\circ$  with the positive direction of the  $x$ -axis as shown.



What is the gradient of the line?

- A.  $\frac{1}{\sqrt{3}}$
- B.  $\frac{1}{\sqrt{2}}$
- C.  $\frac{1}{2}$
- D.  $\frac{\sqrt{3}}{2}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.1	C	0	0	NC	G2	2011 P1 Q8

4. The line through the points  $(-2, 5)$  and  $(7, a)$  has gradient 3.

What is the value of  $a$ ?

- A. 8
- B. 22
- C. 28
- D. 32

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.1	C	0.58	0.16	NC	G2	HSN 05

$$m = \frac{a-5}{7-(-2)} = \frac{a-5}{9} = 3.$$

So  $a-5 = 27$   
 $a = 32$

Option D

5. A line L is perpendicular to the line with equation  $2x - 3y - 6 = 0$ .

What is the gradient of the line L?

- A.  $-\frac{3}{2}$
- B.  $-\frac{1}{2}$
- C.  $\frac{2}{3}$
- D. 2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.1	C	0	0	CN	G2, G5	2010 P1 Q1

6. The line with equation  $y = ax + 4$  is perpendicular to the line with equation  $3x + y + 1 = 0$ .

What is the value of  $a$ ?

- A.  $-3$
- B.  $-\frac{1}{3}$
- C.  $\frac{1}{3}$
- D.  $3$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.1	C	0.7	0.62	NC	G2, G5	HSN 089

$3x + y + 1 = 0$   
 $y = -3x - 1$ . So  $m_1 = -3$ . Compare to  $y = mx + c$

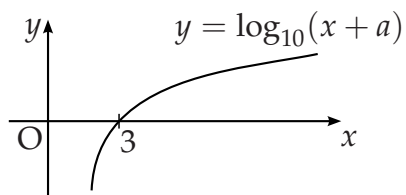
The line  $y = ax + 4$  has gradient  $m_2 = a$

Since the lines are perpendicular,  $m_1 \times m_2 = -1$ , ie

$-3a = -1$   
 $a = \frac{1}{3}$ .

Option C

7. The diagram shows the graph of  $y = \log_{10}(x + a)$ .

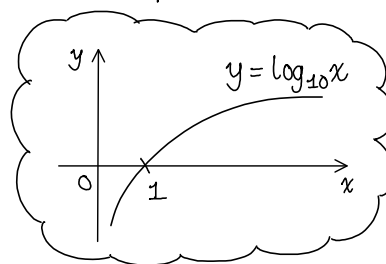


What is the value of  $a$ ?

- A.  $-3$
- B.  $-2$
- C.  $2$
- D.  $3$

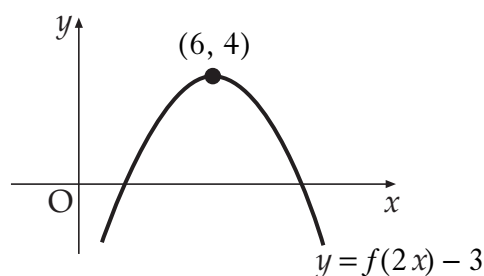
Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.2	C	0.48	0.64	NC	A2, A7	HSN 080

This is the graph of  $y = \log_{10} x$  shifted 2 places to the right. So  $a = -2$ .



Option B

8. The diagram shows the graph of  $y = f(2x) - 3$ .



What are the coordinates of the turning point on the graph of  $y = f(x)$ ?

- A. (12, 7)
- B. (12, 1)
- C. (3, 7)
- D. (3, 1)

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	A/B	0	0	CN	A3	2010 P1 Q20

9. Two functions  $f$  and  $g$  are defined by  $f(x) = 4x + 1$  and  $g(x) = x^2 - 2$ .

Find a formula for  $f(g(x))$ .

- A.  $4x^2 - 7$
- B.  $4x^2 - 1$
- C.  $16x^2 + 8x - 1$
- D.  $4x^3 + x^2 - 8x - 2$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	C	0.69	0.38	CN	A4	HSN 152

$$f(g(x)) = f(x^2 - 2) = 4(x^2 - 2) + 1 = 4x^2 - 8 + 1 = 4x^2 - 7.$$

Option A

10. If  $x^2 - 8x + 7$  is written in the form  $(x - p)^2 + q$ , what is the value of  $q$ ?

- A.  $-9$
- B.  $-1$
- C.  $7$
- D.  $23$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	C	0	0	NC	A5	2011 P1 Q5

11. A function  $f$  is defined by  $f(x) = x^3 + kx^2 + 2x$ .

Given that  $f'(2) = 26$ , what is the value of  $k$ ?

- A.  $3$
- B.  $\frac{7}{2}$
- C.  $5$
- D.  $10$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.3	C	0.65	0.38	NC	C1	HSN 085

$$f(x) = x^3 + kx^2 + 2x$$

$$f'(x) = 3x^2 + 2kx + 2$$

$$f'(2) = 3(2)^2 + 2k \times 2 + 2$$

$$= 12 + 4k + 2$$

$$= 4k + 14.$$

$$\text{So } 4k + 14 = 26$$

$$4k = 12$$

$$k = 3.$$

Option A



12. What is the gradient of the tangent to the curve  $y = 4x^3 + x^2 + 3$  at  $x = 2$ ?

- A.  $24\frac{2}{3}$
- B. 39
- C. 52
- D. 55

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0.53	0.69	NC	C4	HSN 02

$$\frac{dy}{dx} = 12x^2 + 2x.$$

Remember:  
The derivative is the gradient of the tangent.

$$\text{At } x=2, m = 12 \times 2^2 + 2 \times 2 = 12 \times 4 + 4 = 52.$$

Option C

13. A sequence is defined by the recurrence relation  $u_{n+1} = au_n + b$ , where  $a$  and  $b$  are constants.

Given that  $u_0 = 4$  and  $u_1 = 8$ , find an expression for  $a$  in terms of  $b$ .

- A.  $a = \frac{1}{2} - \frac{1}{8}b$
- B.  $a = 2 - \frac{1}{4}b$
- C.  $a = \frac{1}{2} + \frac{1}{8}b$
- D.  $a = 2 + \frac{1}{4}b$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.4	C	0.62	0.5	CN	A10, A14	HSN 060

$$u_0 = 4$$

$$u_1 = a \times 4 + b = 4a + b = 8$$

$$\text{So } 4a = 8 - b$$

$$a = \frac{8}{4} - \frac{b}{4}$$

$$= 2 - \frac{1}{4}b$$

Option B

14. A vector  $v$  is given by  $\begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix}$ .

What is the length, in units, of  $3v$ ?

- A. 7
- B. 15
- C. 21
- D. 49

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.1	C	0.47	0.56	NC	G16	HSN 135

$$\begin{aligned}
 |3v| &= 3|v| = 3\sqrt{(-3)^2 + 2^2 + 6^2} \\
 &= 3\sqrt{9 + 4 + 36} \\
 &= 3\sqrt{49} \\
 &= 3 \times 7 \\
 &= 21.
 \end{aligned}$$

Option C

15. The vectors  $\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$  and  $\begin{pmatrix} k \\ 2 \\ -1 \end{pmatrix}$  are perpendicular.

What is the value of  $k$ ?

- A.  $-3$   
 B.  $3$   
 C.  $\frac{10}{3}$   
 D.  $\frac{8}{3}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	3.1	C	0.63	0.67	NC	G26, G27	HSN 019

$$\begin{aligned} \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} k \\ 2 \\ -1 \end{pmatrix} &= 3k + (-1) \times 2 + 7 \times (-1) \\ &= 3k - 2 - 7 \\ &= 3k - 9 = 0 \quad \text{since the vectors are } \perp \\ \text{So } 3k &= 9 \text{ i.e. } k = 3. \end{aligned}$$

Option B

16. Differentiate  $2(4 - x)^{-\frac{1}{2}}$  with respect to  $x$ .

- A.  $(4 - x)^{-1}$   
 B.  $-(4 - x)^{-1}$   
 C.  $(4 - x)^{-\frac{3}{2}}$   
 D.  $-(4 - x)^{-\frac{3}{2}}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0.52	0.5	NC	C2, C21	HSN 173

$$\begin{aligned} \frac{d}{dx} \left( 2(4 - x)^{-\frac{1}{2}} \right) &= -\frac{1}{2} \times 2 (4 - x)^{-\frac{3}{2}} \times -1 \\ &= (4 - x)^{-\frac{3}{2}}. \end{aligned}$$

Using the Chain rule  
 Option C

17. Find  $\int (2x^{-4} + \cos 5x) dx$ .

- A.  $-\frac{2}{5}x^{-5} - 5 \sin 5x + c$   
 B.  $-\frac{2}{5}x^{-5} + \frac{1}{5} \sin 5x + c$   
 C.  $-\frac{2}{3}x^{-3} + \frac{1}{5} \sin 5x + c$   
 D.  $-\frac{2}{3}x^{-3} - 5 \sin 5x + c$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0	0	CN	C13, C23	2010 P1 Q9

18. Given that  $f(x) = 3 \cos(2x)$ , what is the value of  $f'(\frac{\pi}{6})$ ?

- A. 3  
 B.  $-3\sqrt{3}$   
 C. -3  
 D.  $\frac{3\sqrt{3}}{2}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	3.2	C	0.42	0.59	NC	C20	HSN 017

$$f(x) = 3 \cos(2x)$$

$$f'(x) = 3 \times -2 \sin 2x = -6 \sin 2x.$$

$$f'(\frac{\pi}{6}) = -6 \sin \frac{\pi}{3} = -6 \times \frac{\sqrt{3}}{2} = -3\sqrt{3}.$$

Option B

19. Simplify  $\log_4 8 + \log_4 2 - 3 \log_5 5$ .

- A.  $-\frac{1}{2}$   
 B.  $-1$   
 C.  $\log_4 \left(\frac{16}{5}\right)$   
 D.  $\log_4 \left(\frac{16}{125}\right)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	3.3	A/B	0.53	0.33	NC	A28	HSN 17

$$\begin{aligned}
 \log_4 8 + \log_4 2 - 3 \log_5 5 &= \log_4 8 + \log_4 2 - 3 \times 1 \quad (\log_a a = 1) \\
 &= \log_4 8 \times 2 - 3 \quad (\text{laws of logs}) \\
 &= \log_4 16 - 3 \quad (\text{Since } 4^2 = 16) \\
 &= 2 - 3 \\
 &= -1 \quad \text{Option B}
 \end{aligned}$$

20. Solve  $\log_a 5 + \log_a x = \log_a 20$  for  $x > 0$ .

- A.  $x = \frac{1}{4}$   
 B.  $x = 4$   
 C.  $x = 15$   
 D.  $x = 100$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	3.3	A/B	0.7	0.45	CN	A28, A32	HSN 111

$$\begin{aligned}
 \log_a 5 + \log_a x &= \log_a 20 \\
 \log_a 5x &= \log_a 20 \\
 5x &= 20 \\
 x &= 4
 \end{aligned}$$

$$\begin{aligned}
 &(\log_a x + \log_a y = \log_a xy) \\
 &(\log_a x = \log_a y \Leftrightarrow x = y)
 \end{aligned}$$

Option B

21. Solve  $3 \log_a 2 = \frac{1}{2}$  for  $a$ .

- A.  $a = 64$
- B.  $a = 36$
- C.  $a = \frac{4}{9}$
- D.  $a = \frac{1}{16}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	3.3	A/B	0.56	0.77	CN	A31	HSN 112

$$3 \log_a 2 = \frac{1}{2}$$

$$\log_a 2 = \frac{1}{6}$$

$$a^{\frac{1}{6}} = 2$$

$$a = 2^6$$

$$= 64$$

Changing from log  
to exponential form

Option A

22. The point  $(2, -3)$  lies on the circle with equation  $x^2 + y^2 + 6x - 2y + c = 0$ .

What is the value of  $c$ ?

- A.  $-31$
- B.  $-13$
- C.  $-1$
- D.  $9$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.4	C	0.62	0.57	CN	G10, A6	HSN 065

Let  $(x, y) = (2, -3)$ :

$$2^2 + (-3)^2 + 6(2) - 2(-3) + c = 0$$

$$4 + 9 + 12 + 6 + c = 0$$

$$c = -31.$$

Option A

23. A circle has centre  $(2, 4)$  and passes through  $(-1, 1)$ .

What is the equation of the circle?

- A.  $(x - 2)^2 + (y - 4)^2 = \sqrt{18}$
- B.  $(x - 2)^2 + (y - 4)^2 = 18$
- C.  $(x + 2)^2 + (y + 4)^2 = 18$
- D.  $(x + 2)^2 + (y + 4)^2 = 26$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.4	C	0.51	0.17	NC	G10, G9	HSN 063

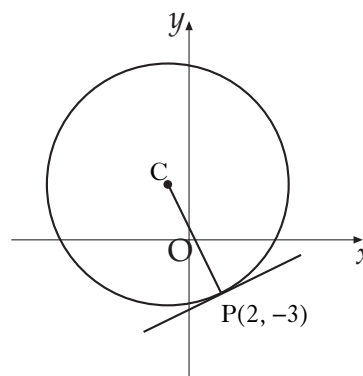
$$r^2 = (2 - (-1))^2 + (4 - 1)^2 = 3^2 + 3^2 = 18.$$

The equation is:  $(x - 2)^2 + (y - 4)^2 = 18$ . Option B

24. The point  $P(2, -3)$  lies on the circle with centre  $C$  as shown.

The gradient of  $CP$  is  $-2$ .

What is the equation of the tangent at  $P$ ?



- A.  $y + 3 = -2(x - 2)$   
 B.  $y - 3 = -2(x + 2)$   
 C.  $y + 3 = \frac{1}{2}(x - 2)$   
 D.  $y - 3 = \frac{1}{2}(x + 2)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.4	C	0	0	NC	G11	2011 P1 Q6

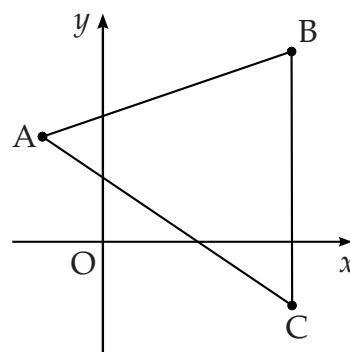
[END OF PAPER 1 SECTION A]



## Paper 1 Section B

25. Triangle ABC has vertices A  $(-3, 5)$ , B  $(9, 9)$  and C  $(9, -3)$ .

- Write down the equation of BC.
- Find the equation of the altitude from A.
- Find the equation of the perpendicular bisector of AB.
- Find where the perpendicular bisector of AB and the altitude from A intersect.



1  
2  
4  
2

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	1	C	CN	G3	$x = 9$	AT077
(b)	2	C	CN	G7	$y = 5$	
(c)	4	C	CN	G7	$3x + y - 16 = 0$	
(d)	2	C	CN	G8	$(\frac{11}{3}, 5)$	

•<sup>1</sup> ic: state equation of vertical line

•<sup>2</sup> ss: use  $m \times m_{\perp} = -1$

•<sup>3</sup> ic: state equation of line

•<sup>4</sup> pd: find gradient of AB

•<sup>5</sup> ss: use  $m \times m_{\perp} = -1$

•<sup>6</sup> ss: find midpoint

•<sup>7</sup> ic: state equation of line

•<sup>8</sup> ss: start to solve equation

•<sup>9</sup> pd: complete

•<sup>1</sup>  $x = 9$

•<sup>2</sup>  $m_{\text{alt.}} = 0$

•<sup>3</sup>  $y = 5$

•<sup>4</sup>  $m_{AB} = 1/3$

•<sup>5</sup>  $m_{\perp} = -3$

•<sup>6</sup>  $\text{midpt}_{AB} = (3, 7)$

•<sup>7</sup>  $y - 7 = -3(x - 3)$

•<sup>8</sup>  $-3x + 16 = 5$

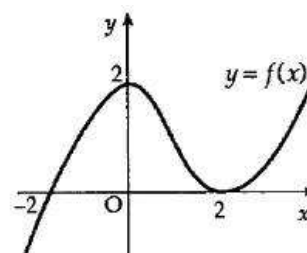
•<sup>9</sup>  $(\frac{11}{3}, 5)$

[SQA] 26. The diagram shows the graph of  $y = f(x)$ , where  $-2 \leq x \leq 3$ .

On separate diagrams, sketch the graphs of

(a)  $y = -f(x)$ ;

(b)  $y = f'(x)$ .



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.4		Source 1991 P1 qu.9
(b)	3	1.2	3						1.2.4		

<ul style="list-style-type: none"> <li>•<sup>1</sup> for correct shape</li> <li>•<sup>2</sup> for annotation</li> <li>•<sup>3</sup> <math>f'(0) = 0</math></li> <li>•<sup>4</sup> <math>f'(2) = 0</math></li> <li>•<sup>5</sup> for correct shape</li> </ul>	
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[SQA] 27.  $f(x) = 2x - 1$ ,  $g(x) = 3 - 2x$  and  $h(x) = \frac{1}{4}(5 - x)$ .

(a) Find a formula for  $k(x)$  where  $k(x) = f(g(x))$ .

(b) Find a formula for  $h(k(x))$ .

(c) What is the connection between the functions  $h$  and  $k$ ?

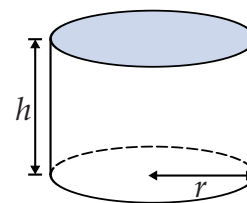
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.6		Source 1993 P1 qu.13
(b)	2	1.2	2						1.2.6		
(c)	1	0.1		1					0.1		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(3 - 2x)</math></li> <li>•<sup>2</sup> <math>5 - 4x</math></li> <li>•<sup>3</sup> <math>h(5 - 4x)</math></li> <li>•<sup>4</sup> <math>x</math></li> <li>•<sup>5</sup> inverse of each other</li> </ul>
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- [SQA] 28. (a) Express  $f(x) = x^2 - 4x + 5$  in the form  $f(x) = (x - a)^2 + b$ . 2
- (b) On the same diagram sketch:
- (i) the graph of  $y = f(x)$ ;
- (ii) the graph of  $y = 10 - f(x)$ . 4
- (c) Find the range of values of  $x$  for which  $10 - f(x)$  is positive. 1

Part	Marks	Level	Calc.	Content	Answer	U1 OC2	
(a)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7	
(b)	4	C	NC	A3	sketch		
(c)	1	C	NC	A16, A6	$-1 < x < 5$		
				<ul style="list-style-type: none"><li>•<sup>1</sup> pd: process, e.g. completing the square</li><li>•<sup>2</sup> pd: process, e.g. completing the square</li><li>•<sup>3</sup> ic: interpret minimum</li><li>•<sup>4</sup> ic: interpret <math>y</math>-intercept</li><li>•<sup>5</sup> ss: reflect in <math>x</math>-axis</li><li>•<sup>6</sup> ss: translate parallel to <math>y</math>-axis</li><li>•<sup>7</sup> ic: interpret graph</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>a = 2</math></li><li>•<sup>2</sup> <math>b = 1</math></li><li>•<sup>3</sup> any two from: parabola; min. t.p. (2, 1); (0, 5)</li><li>•<sup>4</sup> the remaining one from above list</li><li>•<sup>5</sup> reflecting in <math>x</math>-axis</li><li>•<sup>6</sup> translating +10 units, parallel to <math>y</math>-axis</li><li>•<sup>7</sup> (-1, 5) i.e. <math>-1 &lt; x &lt; 5</math></li></ul>		

29. A cylindrical water tank, with solid top and bottom, has radius  $r$  metres and height  $h$  metres. The surface area of the tank is 4 square metres.



- (a) (i) Find an expression for  $h$  in terms of  $r$ .  
 (ii) Hence show that the volume,  $V$  cubic metres, of the tank is given by

$$V = r(2 - \pi r^2).$$

4

- (b) Find the exact value of  $r$  for which the volume  $V$  is a maximum.

5

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	4	C	CN	CGD	$h = \frac{2}{\pi r} - r$ , proof	AT093
(b)	5	C	CN	C11, C3, C8, C9	$r = \sqrt{\frac{2}{3\pi}}$	

<ul style="list-style-type: none"><li>●<sup>1</sup> ss: use area facts</li><li>●<sup>2</sup> pd: process</li><li>●<sup>3</sup> ss: use volume facts</li><li>●<sup>4</sup> ic: complete proof</li><li>●<sup>5</sup> pd: arrange in standard form</li><li>●<sup>6</sup> pd: differentiate</li><li>●<sup>7</sup> ss: set derivative to zero</li><li>●<sup>8</sup> pd: process</li><li>●<sup>9</sup> ic: justification of nature</li></ul>	<ul style="list-style-type: none"><li>●<sup>1</sup> <math>2\pi r^2 + 2\pi rh = 4</math></li><li>●<sup>2</sup> <math>h = \frac{2}{\pi r} - r</math></li><li>●<sup>3</sup> <math>V = \pi r^2 h</math></li><li>●<sup>4</sup> <math>V = r(2 - \pi r^2)</math></li><li>●<sup>5</sup> <math>V = 2r - \pi r^3</math></li><li>●<sup>6</sup> <math>\frac{dV}{dr} = 2 - 3\pi r^2</math></li><li>●<sup>7</sup> <math>\frac{dV}{dr} = 0</math></li><li>●<sup>8</sup> <math>r = \sqrt{\frac{2}{3\pi}}</math></li><li>●<sup>9</sup> <table><tr><td><math>r</math></td><td><math>\rightarrow</math></td><td><math>\sqrt{2/(3\pi)}</math></td><td><math>\rightarrow</math></td></tr><tr><td><math>dV/dr</math></td><td><math>+</math></td><td><math>0</math></td><td><math>-</math></td></tr></table></li></ul>	$r$	$\rightarrow$	$\sqrt{2/(3\pi)}$	$\rightarrow$	$dV/dr$	$+$	$0$	$-$
$r$	$\rightarrow$	$\sqrt{2/(3\pi)}$	$\rightarrow$						
$dV/dr$	$+$	$0$	$-$						

- [SQA] 30. If  $y = x^2 - x$ , show that  $\frac{dy}{dx} = 1 + \frac{2y}{x}$ .

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
3		1.3	1	2					1.3.4	0.1	Source 1989 P1 qu.12
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 2x - 1</math></li> <li>•<sup>2</sup> <math>RHS = 1 + \frac{2(x^2 - x)}{x}</math></li> <li>•<sup>3</sup> <math>1 + 2(x - 1)</math> and complete</li> </ul>											

[SQA] 31. Find  $\frac{dy}{dx}$  where  $y = \frac{4}{x^2} + x\sqrt{x}$ .

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	1.3	4						1.3.4		Source 1995 P1 qu.7
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>4x^{-2}</math> stated or implied by •<sup>3</sup></li> <li>•<sup>2</sup> <math>+x^{\frac{3}{2}}</math> stated or implied by •<sup>4</sup></li> <li>•<sup>3</sup> <math>-8x^{-3}</math></li> <li>•<sup>4</sup> <math>+\frac{3}{2}x^{\frac{1}{2}}</math></li> </ul>											

[SQA] 32. Find the equation of the tangent to the curve  $y = 3x^2 + 2$  at the point where  $x = 1$ .

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	1.3	4						1.3.9	1.1.7	Source 1991 P1 qu.5
<ul style="list-style-type: none"> <li>•<sup>1</sup> strat: <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>f'(1) = 6</math></li> <li>•<sup>3</sup> <math>f(1) = 5</math></li> <li>•<sup>4</sup> <math>y - 5 = 6(x - 1)</math></li> </ul>											

[SQA] 33.

(a) The function  $f$  is defined by  $f(x) = x^3 - 2x^2 - 5x + 6$ .

The function  $g$  is defined by  $g(x) = x - 1$ .

Show that  $f(g(x)) = x^3 - 5x^2 + 2x + 8$ .

4

(b) Factorise fully  $f(g(x))$ .

3

(c) The function  $k$  is such that  $k(x) = \frac{1}{f(g(x))}$ .

For what values of  $x$  is the function  $k$  not defined?

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1 Source 1990 Paper 2 Qu. 6
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	1.2	4						1.2.6		
(b)	3	2.1	3						2.1.3		
(c)	2	1.2	2						1.2.1		

(a)	• <sup>1</sup>	$f(g(x)) = f(x-1)$
	• <sup>2</sup>	$(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$
	• <sup>3</sup>	$(x-1)^3 = x^3 - 3x^2 + 3x - 1$
	• <sup>4</sup>	$-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument
(b)	• <sup>5</sup>	first "0" e.g. $\begin{array}{r rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$
	• <sup>6</sup>	$x^2 - 3x - 4 = (x+1)(x-4)$
	• <sup>7</sup>	$(x-2)(x+1)(x-4)$
(c)	• <sup>8</sup>	denominator $(= (x-2)(x+1)(x-4)) \neq 0$
	• <sup>9</sup>	$-1, 2, 4$

34. The circles centred at A and B have equations  $x^2 + y^2 + 8x + 12y + 36 = 0$  and  $x^2 + y^2 - 4x - 4y - 28 = 0$  respectively.

(a) Write down the coordinates of A and B.

2

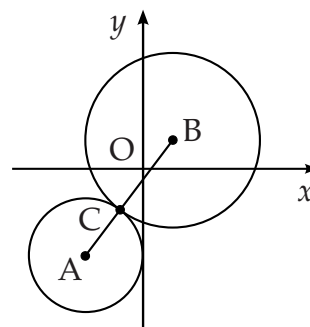
(b) Show that the circles touch externally.

4

(c) The circles touch at point C.

(i) Find the ratio in which C divides AB.

(ii) Hence find the coordinates of C.



4

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G9	A(-4, -6), B(2, 2)	AT085
(b)	4	B	CN	G14	proof	
(c)	4	C	CN	G24, G25, G20	(i) 2 : 3, (ii) C(-8/5, -14/5)	

<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: interpret A</li> <li>•<sup>2</sup> ic: interpret B</li> <li>•<sup>3</sup> pd: find distance between centres</li> <li>•<sup>4</sup> ic: interpret radius</li> <li>•<sup>5</sup> ic: interpret radius</li> <li>•<sup>6</sup> ss: compare sum of radii to distance</li> <li>•<sup>7</sup> ic: state ratio</li> <li>•<sup>8</sup> pd: find vector components</li> <li>•<sup>9</sup> ss: use parallel vectors</li> <li>•<sup>10</sup> pd: process vectors</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> A(-4, -6)</li> <li>•<sup>2</sup> B(2, 2)</li> <li>•<sup>3</sup> AB = 10</li> <li>•<sup>4</sup> rad<sub>A</sub> = 4</li> <li>•<sup>5</sup> rad<sub>B</sub> = 6</li> <li>•<sup>6</sup> AB = rad<sub>A</sub> + rad<sub>B</sub> so touch</li> <li>•<sup>7</sup> 2 : 3</li> <li>•<sup>8</sup> <math>\vec{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}</math></li> <li>•<sup>9</sup> <math>\vec{c} = \vec{a} + \frac{2}{5}\vec{AB}</math></li> <li>•<sup>10</sup> C(-8/5, -14/5)</li> </ul>
--	--

[SQA] 35. Differentiate  $\sin 2x + \frac{2}{\sqrt{x}}$  with respect to  $x$ .

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
4		3.2	2	2					3.2.2	1.3.1	Source 1989 P1 qu.10
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x^{-\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\cos 2x</math></li> <li>•<sup>3</sup> <math>\times 2</math></li> <li>•<sup>4</sup> <math>-x^{-\frac{3}{2}}</math></li> </ul>											

36. The function  $f$ , defined on a suitable domain, is such that  $f'(x) = \frac{1}{\sqrt{(1+x)^3}}$ .

Given that  $f(3) = -1$ , express  $f(x)$  in terms of  $x$ .

5

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	5	B	CN	C18, C22	$-2/\sqrt{1+x}$	AT068
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to integrate</li> <li>•<sup>2</sup> pd: express in integrable form</li> <li>•<sup>3</sup> pd: integrate</li> <li>•<sup>4</sup> ic: substitute values</li> <li>•<sup>5</sup> pd: process</li> </ul>						
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int f'(x) dx</math></li> <li>•<sup>2</sup> <math>f'(x) = (1+x)^{-3/2}</math></li> <li>•<sup>3</sup> <math>-2(1+x)^{-1/2} + c</math></li> <li>•<sup>4</sup> <math>-2/\sqrt{4} + c = -1</math></li> <li>•<sup>5</sup> <math>c = 0</math></li> </ul>						

[SQA] 37. Given  $x = \log_5 3 + \log_5 4$ , find algebraically the value of  $x$ .

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
4		3.3	1	3					3.3.3	3.3.1, 3.3.4	Source 1998 P1 qu.19
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x = \log_5 12</math></li> <li>•<sup>2</sup> <math>5^x = 12</math></li> <li>•<sup>3</sup> <math>\log 5^x = \log 12</math></li> <li>•<sup>4</sup> <math>\frac{\log_{10} 12}{\log_{10} 5}</math> or <math>\frac{\log_e 12}{\log_e 5}</math> or <math>\frac{\log 12}{\log 5} = 1.54</math></li> </ul>											



[SQA] 38. Circle P has equation  $x^2 + y^2 - 8x - 10y + 9 = 0$ . Circle Q has centre  $(-2, -1)$  and radius  $2\sqrt{2}$ .

(a) (i) Show that the radius of circle P is  $4\sqrt{2}$ .

(ii) Hence show that circles P and Q touch.

4

(b) Find the equation of the tangent to the circle Q at the point  $(-4, 1)$ .

3

(c) The tangent in (b) intersects circle P in two points. Find the  $x$ -coordinates of the points of intersection, expressing your answers in the form  $a \pm b\sqrt{3}$ .

3

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	2	C	CN	G9	proof	2001 P1 Q11
(a)	2	A/B	CN	G14		
(b)	3	C	CN	G11	$y = x + 5$	
(c)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

- <sup>1</sup> ic: interpret centre of circle (P)
- <sup>2</sup> ss: find radius of circle (P)
- <sup>3</sup> ss: find sum of radii
- <sup>4</sup> pd: compare with distance between centres

- <sup>5</sup> ss: find gradient of radius
- <sup>6</sup> ss: use  $m_1 m_2 = -1$
- <sup>7</sup> ic: state equation of tangent

- <sup>8</sup> ss: substitute linear into circle
- <sup>9</sup> pd: express in standard form
- <sup>10</sup> pd: solve (quadratic) equation

- <sup>1</sup>  $C_P = (4, 5)$
- <sup>2</sup>  $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
- <sup>3</sup>  $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
- <sup>4</sup>  $C_P C_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$  and "so touch"

- <sup>5</sup>  $m_r = -1$
- <sup>6</sup>  $m_{\text{tgt}} = +1$
- <sup>7</sup>  $y - 1 = 1(x + 4)$

- <sup>8</sup>  $x^2 + (x + 5)^2 - 8x - 10(x + 5) + 9 = 0$
- <sup>9</sup>  $2x^2 - 8x - 16 = 0$
- <sup>10</sup>  $x = 2 \pm 2\sqrt{3}$

39. The diagram below shows the graph of the cubic with equation  $y = x^3 - 3x^2 + 5x + 4$  and a circle with centre C.

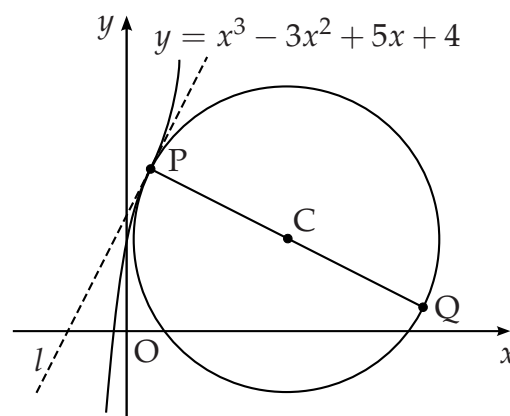
At the point P the line  $l$  is a tangent to both the curve and the circle.

- (a) The tangent line  $l$  has gradient 2.  
Find the coordinates of P.

- (b) The circle has equation  
 $x^2 + y^2 - 14x - 8y + c = 0$ .

Determine the value of  $c$ .

- (c) The line PQ is a diameter of the circle. Determine the coordinates of Q.



5

2

2

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	5	C	CN	C4	P(1,7)	AT059
(b)	2	C	CN	G10	$c = 20$	
(c)	2	C	CN	G9, G6	Q(13,1)	

- <sup>1</sup> ss: know to differentiate
- <sup>2</sup> pd: differentiate
- <sup>3</sup> ss: equate
- <sup>4</sup> pd: process solution
- <sup>5</sup> ic: complete coordinates

- <sup>6</sup> ss: substitute
- <sup>7</sup> pd: process

- <sup>8</sup> ic: interpret coordinates
- <sup>9</sup> ss: use midpoint relationship

- <sup>1</sup>  $dy/dx = \dots$
- <sup>2</sup>  $\dots = 3x^2 - 6x + 5$
- <sup>3</sup>  $3x^2 - 6x + 5 = 2$
- <sup>4</sup>  $x = 1$
- <sup>5</sup> P(1,7)

- <sup>6</sup>  $1 + 7^2 - 14 - 8 \times 7 + c = 0$
- <sup>7</sup>  $c = 20$

- <sup>8</sup> C(7,4)
- <sup>9</sup> Q(13,1)

[END OF PAPER 1 SECTION B]

## Paper 2

1. The function  $f$  is defined by  $f(x) = x^3 + px^2 + qx + 3$ .

The tangent to the curve  $y = f(x)$  at  $x = 1$  has gradient  $-3$ .

(a) Show that  $2p + q = -6$ .

3

(b) Given that 3 is a root of the equation  $f(x) = 0$ , find the values of  $p$  and  $q$ .

4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	3	C	CN	C4	proof	AT038
(b)	4	C	CN	A6	$p = -4, q = 2$	

- <sup>1</sup> ss: know to differentiate
- <sup>2</sup> ic: interpret data
- <sup>3</sup> ic: complete
- <sup>4</sup> ic: interpret data
- <sup>5</sup> ss: start to solve simultaneously
- <sup>6</sup> pd: solve for one variable
- <sup>7</sup> pd: solve for second variable

- <sup>1</sup>  $f'(x) = 3x^2 + 2px + q$
- <sup>2</sup>  $f'(1) = -3$
- <sup>3</sup>  $2p + q = -6$
- <sup>4</sup>  $9p + 3q = -30$
- <sup>5</sup>  $2p + q = -6, 9p + 3q = -30$
- <sup>6</sup>  $p = -4$
- <sup>7</sup>  $q = 2$

- [SQA] 2. A sequence is defined by the recurrence relation  $u_n = 0.9u_{n-1} + 2, u_1 = 3$ .

(a) Calculate the value of  $u_2$ .

1

(b) What is the smallest value of  $n$  for which  $u_n > 10$ ?

1

(c) Find the limit of this sequence as  $n \rightarrow \infty$ .

2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	1.4			1				1.4.2		Source 1994 P1 qu.9
(b)	1	1.4			1				1.4.3		
(c)	2	1.4			2				1.4.5		

•<sup>1</sup> 4.7

•<sup>2</sup> 7

•<sup>3</sup>  $l = 0.9l + 2$

OR

•<sup>3</sup>  $l = \frac{b}{1-a} = \frac{2}{1-0.9}$

•<sup>4</sup> 20

•<sup>4</sup> 20

- [SQA] 3. (a) At 12 noon a hospital patient is given a pill containing 50 units of antibiotic.  
 By 1 pm the number of units in the patient's body has dropped by 12%.  
 By 2 pm a further 12% of the units remaining in the body at 1 pm is lost.  
 If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6 pm. (4)
- (b) A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time.  
 The doctor knows that more than 100 units of this antibiotic in the body is regarded as too dangerous.  
 Should the doctor prescribe this course of treatment?  
 Give reasons for your answer. (6)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	1.4			4				1.4.1		Source 1991 Paper 2 Qu. 9
(b)	6	1.4			4	2			1.4.3, 1.4.5		

- (a)
- <sup>1</sup> use 0.88 or 88%
  - <sup>2</sup>  $n = 6$
  - <sup>3</sup>  $u_6 = 50 \times 0.88^6$
  - <sup>4</sup> 23.22
- (b)
- <sup>5</sup> adding 50
  - <sup>6</sup>  $u_{n+1} = 0.88^6 u_n + 50$
  - <sup>7</sup>  $-1 < 0.88^6$  (or 0.4644)  $< 1$  so limit exists
  - <sup>8</sup>  $L = \frac{50}{1 - 0.88^6}$
  - <sup>9</sup> 93.4
  - <sup>10</sup>  $93.4 < 100$  so safe to continue

[SQA] 4. ABCD is a quadrilateral with vertices  $A(4, -1, 3)$ ,  $B(8, 3, -1)$ ,  $C(0, 4, 4)$  and  $D(-4, 0, 8)$ .

- (a) Find the coordinates of M, the midpoint of AB. 1
- (b) Find the coordinates of the point T, which divides CM in the ratio 2 : 1. 3
- (c) Show that B, T and D are collinear and find the ratio in which T divides BD. 4

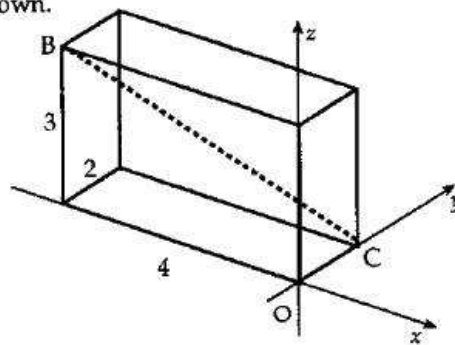
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1 Source 1989 Paper 2 Qu. 2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	0.1					1		0.1		
(b)	3	3.1					3		3.1.6		
(c)	4	3.1					4		3.1.7, 3.1.6		

- (a) •<sup>1</sup>  $(6, 1, 1)$
- (b) •<sup>2</sup> e.g.  $\vec{CM} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$
- <sup>3</sup>  $\vec{CT} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$
- <sup>4</sup>  $T = (4, 2, 2)$
- (c) •<sup>5</sup> e.g.  $\vec{BT} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$
- <sup>6</sup>  $\vec{TD} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} = 2 \times \vec{BT}$
- <sup>7</sup> TD is parallel to BT, T is common point so B, T, D collinear
- <sup>8</sup> BT:TD = 1:2

[SQA] 5. A cuboid crystal is placed relative to the coordinate axes as shown.

(a) Write down  $\vec{BC}$  in component form.

(b) Calculate  $|\vec{BC}|$ .



2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	3.1					1		3.1.1		Source
(b)	1	3.1					1		3.1.3		1990 P1 qu.5

1  $\vec{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

2  $\sqrt{29}$

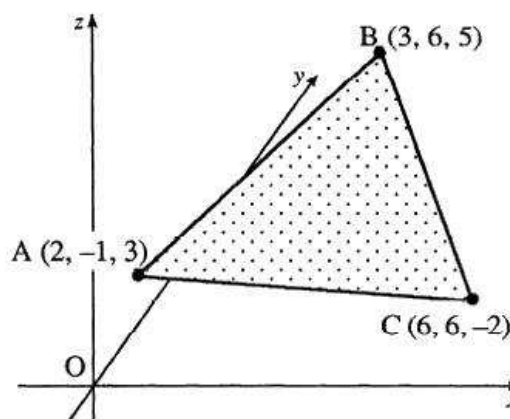
[SQA]

6.

A triangle ABC has vertices

A (2, -1, 3), B(3, 6, 5) and C (6, 6, -2).

- (a) Find  $\vec{AB}$  and  $\vec{AC}$ .  
 (b) Calculate the size of angle BAC.  
 (c) Hence find the area of the triangle.



(2)

(5)

(2)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	3.1			2				3.1.1		Source 1998 Paper 2 Qu. 1
(b)	5	3.1			5				3.1.11		
(c)	2	0.1			2				0.1		

(a)	• <sup>1</sup>	$\vec{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$									
	• <sup>2</sup>	$\vec{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$									
(b)	• <sup>3</sup>	$\cos \hat{BAC} = \frac{\vec{AB} \cdot \vec{AC}}{ \vec{AB}   \vec{AC} }$	stated or implied by responses to • <sup>4</sup> to • <sup>7</sup>								
	• <sup>4</sup>	$\vec{AB} \cdot \vec{AC} = 4 + 49 - 10$									
	• <sup>5</sup>	$ \vec{AB}  = \sqrt{54}$									
	• <sup>6</sup>	$ \vec{AC}  = \sqrt{90}$									
	• <sup>7</sup>	$\hat{BAC} = 51.9^\circ$									
	(c)	• <sup>8</sup>	identify 2 sides and included angle e.g. $\sqrt{54}$ , $\sqrt{90}$ , $\hat{BAC}$								
		• <sup>9</sup>	$27 \cdot 4$								



- [SQA] 7. VABCD is a pyramid with rectangular base ABCD.

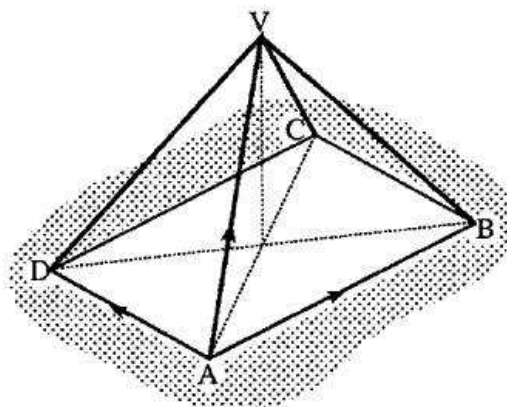
The vectors  $\vec{AB}$ ,  $\vec{AD}$  and  $\vec{AV}$  are given by

$$\vec{AB} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\vec{AD} = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k} \quad \text{and}$$

$$\vec{AV} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}.$$

Express  $\vec{CV}$  in component form.



3

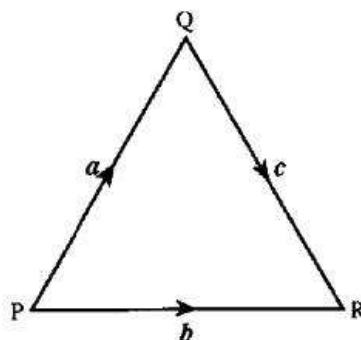
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
	3	3.1					3		3.1.8		Source 1999 P1 qu.6
$\bullet^1$ pathway for $\vec{CV}$ : $\vec{CV} = \vec{CA} + \vec{AV}$ $\bullet^2$ e.g. $\vec{CB} = 2\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}$ or $\vec{BA} = -8\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ or $\vec{AC} = 6\mathbf{i} + 12\mathbf{j}$											

$$\bullet^3 \begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$$

- [SQA] 8. PQR is an equilateral triangle of side 2 units.

$$\vec{PQ} = \mathbf{a}, \vec{PR} = \mathbf{b} \quad \text{and} \quad \vec{QR} = \mathbf{c}.$$

Evaluate  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$  and hence identify two vectors which are perpendicular.



4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
	4	3.1					1	3	3.1.9	3.1.1	Source 1997 P1 qu.13
$\bullet^1$ $\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ $\bullet^2$ $\mathbf{a} \cdot \mathbf{b} = 2 \times 2 \times \frac{1}{2}$ $\bullet^3$ $\mathbf{a} \cdot \mathbf{c} = 2 \times 2 \times -\frac{1}{2}$ $\bullet^4$ 0 and $\mathbf{a}$ is perpendicular to $(\mathbf{b} + \mathbf{c})$											



- [SQA] 9. (a) A tractor tyre is inflated to a pressure of 50 units.  
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure,  $P_t$  units, after  $t$  hours is given by the formula  $P_t = P_0 e^{-kt}$ , find the value of  $k$ , to three decimal places.

(5)

- (b) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre?

(4)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	5	3.3			2	3			3.3.4		Source 1991 Paper 2 Qu. 7
(b)	4	3.3			1	3			3.3.4		

- (a)
- <sup>1</sup>  $10 = 50e^{-24k}$
  - <sup>2</sup>  $0.2 = e^{-24k}$
  - <sup>3</sup>  $-24k = \ln 0.2$
  - <sup>4</sup>  $-24k = -1.609$
  - <sup>5</sup>  $k = 0.067$
- (b)
- <sup>6</sup> knowing to find  $P_4$
  - <sup>7</sup>  $P_4 = 50e^{-0.067 \times 4}$
  - <sup>8</sup> 38
  - <sup>9</sup>  $38 > 30$  so can be driven further

- [SQA] 10. (a) Express  $\sin x^\circ - 3 \cos x^\circ$  in the form  $k \sin(x - a)^\circ$  where  $k > 0$  and  $0 \leq a < 360$ . Find the values of  $k$  and  $a$ .

4

- (b) Find the maximum value of  $5 + \sin x^\circ - 3 \cos x^\circ$  and state a value of  $x$  for which this maximum occurs.

2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	3.4			4				3.4.1		Source 1992 P1 qu.7
(b)	2	3.4			1	1			3.4.3		

$\bullet^1 k \cos a = 1$ 
 $\bullet^5 \text{ maximum} = 5 + \sqrt{10}$   
 $\bullet^2 k \sin a = 3$ 
 $\bullet^6 \text{ angle} = 161.6^\circ$   
 $\bullet^3 k = \sqrt{10}$   
 $\bullet^4 a = 71.6$

- [SQA] 11.

- (a) Show that  $2 \cos(x^\circ + 30^\circ) - \sin x^\circ$  can be written as  $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$ .

3

- (b) Express  $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$  in the form  $k \cos(x^\circ + \alpha^\circ)$  where  $k > 0$  and  $0 \leq \alpha \leq 360$  and find the values of  $k$  and  $\alpha$ .

4

- (c) Hence, or otherwise, solve the equation  $2 \cos(x^\circ + 30^\circ) = \sin x^\circ + 1$ ,  $0 \leq x \leq 360$ .

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	3	2.3			3				2.3.2,	1.2.11	Source 1990 Paper 2 Qu. 5
(b)	4	3.4			4				3.4.1		
(c)	3	3.4				3			3.4.2		

$\bullet^1 \cos(x + 30)^\circ = \cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$   
 $\bullet^2 \frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ$   
 $\bullet^3 2 \times \left( \frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ \right) - \sin x^\circ$   
  
 $\bullet^4 k \cos x^\circ \cos \alpha^\circ - k \sin x^\circ \sin \alpha^\circ$   
 $\bullet^5 k \sin \alpha^\circ = \sqrt{3} \text{ and } k \sin \alpha^\circ = 1$   
 $\bullet^6 k = \sqrt{7} \vec{OG} = 426$   
 $\bullet^7 \alpha = 49.1$   
  
 $\bullet^8 \sqrt{7} \cos(x + 49.1)^\circ = 1$   
 $\bullet^9 x = 18.7^\circ$   
 $\bullet^{10} x = 243.1^\circ$

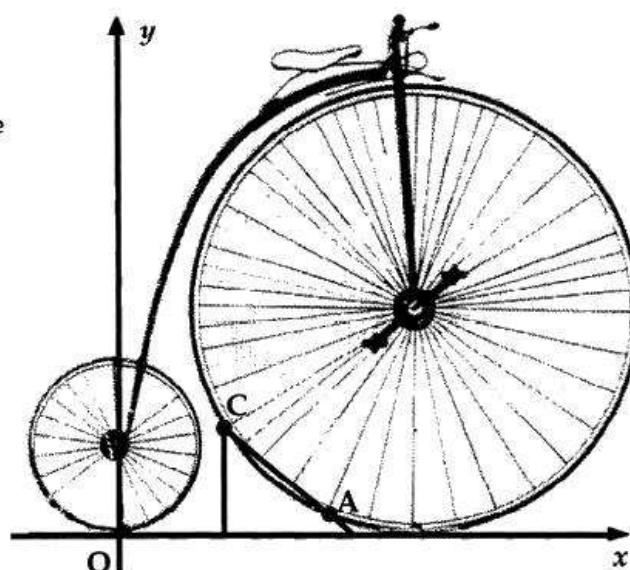
- [SQA] 12. A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.

With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is

$$x^2 + y^2 - 6y = 0 \text{ and}$$

the equation of the front wheel is

$$x^2 + y^2 - 28x - 20y + 196 = 0.$$



- (a) (i) Find the distance between the centres of the two wheels.  
 (ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. (8)
- (b) B(7,3) is half-way between A and C, and P is the centre of the front wheel.  
 (i) Find the gradient of PB.  
 (ii) Hence find the equation of AC and the coordinates of A and C. (8)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	8	2.4			8				2.4.2,	1.1.2	Source 1994 Paper 2 Qu.4
(b)	8	1.1			8				1.1.1,	1.1.9, 2.4.4	

- (a)
- <sup>1</sup> centre (0, 3)
  - <sup>2</sup> centre (14, 10)
  - <sup>3</sup> distance between centres =  $\sqrt{245}$
  - <sup>4</sup> radius = 3
  - <sup>5</sup> radius = 10
  - <sup>6</sup> strategy (clearance = distance between centres minus sum of radii)
  - <sup>7</sup>  $\sqrt{245} - 13$
  - <sup>8</sup> 133 mm or equivalent

- (b)
- <sup>9</sup>  $m_{PB} = 1$
  - <sup>10</sup>  $m_{AC} = -1$
  - <sup>11</sup>  $y - 3 = -(x - 7)$  for AC
  - <sup>12</sup> strategy: substitute
  - <sup>13</sup> substituting correctly
  - <sup>14</sup> eg  $2x^2 - 28x + 96 = 0$
  - <sup>15</sup>  $x = 6, 8$  (or  $y = 2, 4$ )
  - <sup>16</sup> (6, 4) and (8, 2)

[SQA] 13. Solve the equation  $3 \cos 2x^\circ + \cos x^\circ = -1$  in the interval  $0 \leq x \leq 360$ .

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	60, 131.8, 228.2, 300	2000 P2 Q5
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to use</li> <li><math>\cos 2x = 2 \cos^2 x - 1</math></li> <li>•<sup>2</sup> pd: process</li> <li>•<sup>3</sup> ss: know to/and factorise quadratic</li> <li>•<sup>4</sup> pd: process</li> <li>•<sup>5</sup> pd: process</li> </ul>				<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3(2 \cos^2 x^\circ - 1)</math></li> <li>•<sup>2</sup> <math>6 \cos^2 x^\circ + \cos x^\circ - 2 = 0</math></li> <li>•<sup>3</sup> <math>(2 \cos x^\circ - 1)(3 \cos x^\circ + 2)</math></li> <li>•<sup>4</sup> <math>\cos x^\circ = \frac{1}{2}, x = 60, 30</math></li> <li>•<sup>5</sup> <math>\cos x^\circ = -\frac{2}{3}, x = 132, 228</math></li> </ul>		

[END OF PAPER 2]